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MATHEMATICAL MODEL OF BATCH SYSTEM ON E. COLI MIGRATION ON HOMOGENEOUS FINE SAND INFLUENCED BY POROSITY IN CAOSTAL AREA OF PORT HARCOURT

Eluozo, S. N.

Subaka Nigeria Limited Port Harcourt Rivers State of Nigeria
Director and principal consultant Civil and Environmental Engineering, Research and Development
E-mail: Soloeluozo2013@hotmail.com
E-mail: solomoneluozo2000@yahoo.com

Abstract

Environmental and public health problems are mostly associated with indiscriminate dumping of waste and poor environmental sanitation, this has caused lots of illnesses to human, and several illnesses are generated from poor sanitation in the study area. Ground water pollution has been an issue of environmental concern, because it has caused lots of deaths in the study area. The area is a deltaic environment; this study location generates lots of environmental influences coupled with the geological setting of the area, which influences the migration of the microbes to ground water aquifer. The system application were considered since the soil deposits homogeneous formation, other influences like climatic condition that develop high rain intensities, the variables were considered before mathematical equations were developed. The equations were derived and generate a model to monitor the migration on batch system application in homogenous fine sand; this is under the influence of porosity of the soil. Experts will find this developed model useful in the management of ground water system.

Keywords: Mathematical model, batch system , e. coli migration on homogeneous fine sand, and porosity

1. Introduction

Escherichia coli are a common inhabitant of the gastrointestinal tract of humans and animals. There are *E. coli* strains that are harmless commensal of the intestinal tract and others that are major pathogens of humans and animals. The pathogenic *E. coli* is divided into those strains causing disease inside the intestinal tract and others capable of infection at extra intestinal sites (Kaper et al., 2004). *Escherichia coli* is easily cultured in the clinical laboratory, but the identification of the different pathogenic genotypes requires virulence gene detection methods. *Escherichia coli* can be found secondarily in soil and water as a result of fecal contamination. Classically, its detection has been used as an indicator of poor water and/or food quality. From biochemical, physiological and genetic perspectives, *E. coli* is one of the best understood and characterized living organisms, with laboratory studies on model strains such as *E. coli* K-12 taking place over the past sixty years.

Infections due to pathogenic *E. coli* may be limited to colonization of a mucosal surface or can disseminate throughout the body and have been implicated in urinary tract infection, sepsis/meningitis and gastrointestinal infections (Nataro&Kaper, 1998). Due to the ease of access of pathogens ingested with food, the human gastro-intestinal tract is susceptible to diarrhoeagenic *E. coli* infections. Several *E. coli* pathotype have been implicated with diarrheal illness, a major public health problem worldwide, with over two million deaths occurring each year (Kosek et al., 2003).

This increasing concern about water quality and quantity necessitates the interventions in water systems to meet the objective of sustainable water supply and prevent potential environmental deterioration. Zacharias et al.,(2005), Francisca (2010) emphasized that sustainable water management which incorporates both socio-economic and environmental perspectives is a difficult but essential task in order to prevent potential environmental deterioration. In recent years the large amounts of polluted water are discharged into rivers and causing serious future uncertainty in the water quality. However, method that integrates water quantity and quality in water resource allocation has the potential to add value to decision makers who face these challenges (Zhang et al., 2010). Various conventional methods are in practice for purification of water and removing the pollutant contaminants, but most of them are costly and non-ecofriendly (Dhote and Dixit, 2008 Francisca 2010).

The disposal of metropolitan solid waste (MSW) has the prospective to impact the environment pessimistically. The main concern is to avoid the pollution of soil and water by the leachate that originates in the decay of the solid waste inside landfills (Kjeldsen et al., 2002). The quantity and chemical composition of leachate are determined on the water that infiltrates in the landfill, and on the substance reactions among the solid and liquid phases, including disbanding, precipitation, ion exchange and biochemical processes. Leachate migration from inside the landfill cell to the vadose zone is prohibited by low permeability liners (Petrov and Rowe, 1997; Guyonnet et al., 2005; Touze-Foltz et al., 2006), which usually have multiple layers of compacted clay, granular filters and geosynthetics. Compacted clays or mixtures of local soils with clay are frequently used to achieve very low hydraulic conductivity barriers and prevent subsurface contamination. The hydraulic conductivity can be further reduced by the addition of Bentonite to local soils to attain the values specified by international regulations (kb10−7 cm/s) (Kayabali, 1997; Goldman et al., 1998). The ability of compacted soil liners to restrict the movement of water and contaminants depends on particle size, void ratio, specific surface, degree of saturation, and fluid properties (Vuković and Soro, 1992; Foged and Baumann, 1999). Soil fabric, compaction energy and thixotropy are also relevant properties (Daniel and Benson, 1990; Benson and Trast, 1995). Different particle associations created during compaction generate either flocculated or dispersed soil fabrics, and are of fundamental importance in the soil hydraulic conductivity (Mitchell et al., 1965). In the past two decades, several studies were conducted to evaluate how soil and liquid properties control the hydraulic conductivity of soil liners (Mitchell et al., 1965; Mitchell and Jaber, 1990; Gleason et al., 1997; Schmitz, 2006). In general, the hydraulic conductivity of soils decreases with increasing fine particle content (Sivapullaiah et al., 2000). At high mechanical stress levels and in the case of highly compacted soils, electrical forces have negligible effect on soil behavior and soil fabric is slightly affected by the chemical properties of the permeating liquid (Mitchell and Soga, 2005). However, hydraulic behavior of fine soils with high porosity and freshly compacted soils is highly influenced by the interaction between the pore fluid and mineral particles. Another important property is the retention capacity of the soil, which depends on adsorption mechanisms that delay the passage of contaminants through soil liners. The adsorption of the ions present in the permeating liquid by the mineral surface is controlled by surface charge density of the particles, pH, ion concentration, ion valence, dielectric permittivity and temperature of the pore fluid (Clement et al., 1998; Aringhieri and Giachetti, 2001). The counter ions in the double layer can be replaced by other hydrated cations while electroneutrality is preserved, increasing the residence time of contaminant species within soil liners (Fetter, 1993). Adsorption

2. Theoretical background

The behaviour of microbes that deposit in a particular region were monitored to determine their rate of concentration and migration within the uniform formation. This condition implies that the microbes are not going to migrate to any other formation but uniform soil strata. Formation characteristics in this study that play the major role is porosity considered in the system, this is because uniform fine sand is porous in deltaic environment. Such environment develops a higher rate of concentration of E.coli in such homogeneous formation. There are lots of areas in deltaic environment that deposit such homogenous formation, reacting to this condition, a batch system behaviour were applied in such a uniform formation to monitor the migration of microbes in the system.These formations were assumed not to deposit any other characteristics that will dominate porosity in the system; this implies that porosity is the major influence on the microbial transport process. Such deposition implies that E.coli transport may develop an increase in concentration to an optimum level if there is no inhibition from any deposited mineral.

Groundwater qualities in such kind of homogeneous formation are prone to develop high concentration if there regeneration of such microbial deposition in the environment, but if there is no regeneration of such microbial deposition the concentration of the microbes will reduce. Therefore, the deposition of the microbes are determined by the rate of biological waste dump in the environment, the behaviuor of the system are determined by the rate of pollution generated in the environment, indiscriminate dumping of waste has resulted to increase in such microbial concentration leaching to ground water aquifers. The behaviour of the microbe in such a homogeneous formation was considered to develop a mathematical model that will monitor the behaviuor of the microbes in homogeneous formation

3 Governing Equation

$$K\,C_{(x)}\,\frac{\partial V_{(x)}}{\partial t} = \frac{V\partial C_{(x)}}{\partial t} \qquad \text{....................................} \qquad (1)$$

The governing equation is an expression developed to monitor the rate of E.coli deposition in a homogeneous fine soil, these mathematical equations were developed considering some deltaic environments that deposits homogeneous soil formation in deltaic environment. The equations were developed considering the influential variables that played a major role in the system. the variables that can monitor the concentration in such environment are stated in equation (1) above.

$$\frac{V\partial C_{(x)}}{\partial t} = K\,C_{(x)}\frac{\partial V_{(x)}}{\partial t} \qquad \text{.......................................} \qquad (2)$$

$$\frac{V\partial C_{(x)}}{\partial t} = -K\,C_{(x)}\frac{V_x}{t} \qquad \text{.......................................} \qquad (3)$$

$$\left(\frac{V}{V_x}\right)\frac{\partial C_{(x)}}{\partial (x)} = -\frac{Kdt}{t} \qquad \text{.......................................} \qquad (4)$$

$$V/V = \int 1/C_{(x)}\ \partial C_{(x)} = -K\int\frac{\partial t}{t} \qquad \text{.......................................} \qquad (5)$$

$$V/V_{(x)}\left[\ln C_{(x)} = -K\ln\frac{t_o}{t}\right] \qquad \text{.......................................} \qquad (6)$$

$$\ln\frac{C_{(x)}}{C_{(x)_o}} = -K\frac{V_{(x)}}{V}\ln\frac{t}{t_o} = \ln\left(\frac{t}{t_o}\right) - K\frac{V_x}{V} \qquad \text{.......................................} \qquad (7)$$

$$\frac{C_{(x)}}{C_{(x)_o}} = \left(\frac{t}{t_o}\right) - \frac{KV_x}{V} \qquad \text{.......................................} \qquad (8)$$

$$\frac{C_{(x)}}{C_{(x)_o}} = \ell^{-K\ln\left(t/t_o\right)V_x/V} \qquad \text{.......................................} \qquad (9)$$

$$C_{(x)} = C_{(x)_o}\,\ell^{-K\ln 1/t\ V_x/V} \qquad \text{.......................................} \qquad (10)$$

$$C_{(x)} = \beta\,\ell^{-K\ln 1/t}\ Vx/V \qquad \text{.......................................} \qquad (11)$$

$$\boxed{\beta = C_{(x)_o}\,\ell\frac{V_{(x)}}{tv}} \qquad \text{.......................................} \qquad (12)$$

The developed equations were derived to express the various variables in terms of their functions in the system.The derivation generated a model as expressed in (12), whereby concentrations with respect to time were expressed under the velocity of transport in the system. The derived model in this system implies that in such homogenous formation on fine sand, velocity of flow will be higher under constant rate. Microbial deposition in such soil formation can only deposit low concentration when micro element are not found in such environment, indiscriminate dumping of biological wastes are the challenges in such environment under the influence of manmade activities. The chance of not depositing E.coli in the environment becomes very slim because there is the tendency of waste generation in the environment that is found to deposit homogeneous strata. The action from manmade activities is a subject of environmental concern on public health because the major generations of E.coli from biological wastes are caused by untreated wastes in the environment.

The model can be applied to monitor the rate of E. coli transport on a batch system influenced by porosity.

Integrating the parameter in the equation it becomes

$$C_{(x)} = \beta \ell^{nVt} \qquad \text{.. (13)}$$

Taking Laplace transform of (13), gives

$$C_{(o)} = \frac{\beta}{nV+S} \qquad \text{...................................... (14)}$$

$$\Rightarrow \quad C_{(o)}\left[nV+S\right] = \beta$$

$$C_{(o)}\, nV + C_{(o)}\, S - \beta = 0 \qquad \text{...................................... (15)}$$

Applying quadratic expression on (15), it becomes

$$C_{(x)} = \frac{-S \pm \sqrt{S^2 + 4}\beta nV}{2nV} \qquad \text{...................................... (16)}$$

Further expression in the system were to descretize some other variables that expressed their influence in the system, quadratic expressions were applied. The equation expressed the role of other variables by integrating the independent variable applied in the system; this is to streamline the behaviour of lack of concentration under degradation and exponential level on transport process in a homogeneous soil formation. The applications of quadratic expression definitely express the behaviour of E.coli transport process in this phase.

Now $S = nV$ substitute it in equation (16) so that we can have

$$C_{(x)} = \frac{-nV \pm \sqrt{n^2V^2 + 4}\beta nV}{2nV} \qquad \text{...................................... (17)}$$

Therefore, the general solution can be expressed as

$$C_{(x)} = A\exp\left[\frac{-nV+\sqrt{n^2V^2+4\beta nV}}{2nV}\right]t + \exp\left[\frac{-nV+\sqrt{n^2V^2+4\beta nV}}{2nV}\right]t \quad \ldots\ldots\ldots \quad (18)$$

Subjecting (18) to the following boundary and initial conditions:

At $x = 0$, $C_{(o)} = 0$ and $t = 0$, so that we can have

$0 = A + \beta$, i.e. $A = -\beta$

If $A = 1 \Rightarrow \beta = -1$

and our equation (18) can be expressed thus,

$$C_{(x)} = \exp\left[\frac{-nV+\sqrt{n^2V^2+4\beta nV}}{2nV}\right]t - \exp\left[\frac{-nV-\sqrt{n^2V^2+4\beta nV}}{2nV}\right]t \quad \ldots\ldots\ldots\ldots \quad (19)$$

Thus, we convert $\dfrac{\ell^x - \ell^{-x}}{2} = Sin x$

So that we can rewrite our equation (19) as

$$C_{(x)} = 2\,Sin x\left[\frac{nV+\sqrt{n^2V^2+4\beta nV}}{2nV}\right]t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

The expression at (20) considering all the variables in the system, the developed model expressed all the variables that influence the transport system of E.coli in homogenous fine formation, deltaic environment are found to deposit uniform soil stratification. Therefore, the batch system was applied in developing the mathematical equation that governs the migration of E.coli in homogenous fine sand. The influence of porosity was considered in such homogeneous formation in the study area because other formation characteristics were assumed to be inactive in such environment that deposits homogeneous soil formation. Such environmental influence in homogeneous formation develops rapid microbial growth if there is deposition of micro element in the study area.

4. Conclusion

Formation characteristics such as porosity of the soil played major role in the migration of E.coli in homogeneous fines sand, the study was to monitor the rate of E.coli concentration in a homogeneous fine sand. Degree of porosity in fine sand formation are known to develop high percentage, therefore, porosity determine the rate of E.coli migration in such deposited formation. Batch systems were applied on the development of the model; the expressions were derived under this condition to monitor the rate of concentration at a uniform fine sand formation.

This condition has been a major threat to the settlers in the environment because from the hydro geological studies, the formation deposits shallow aquifers, the hydrological condition expressed several pollution factors that increase concentration of E.coli deposition in ground water aquifers. Environmental factors were considered in the transport of E.coli in homogeneous soil formation because the environment has climatic condition that has high rain intensities. This dimension has some influence son the deposition of the formation, whereby high rain intensities increase the degree of soil saturation and ground water level. So at shallow aquifers the tendency of leaching fast through the saturation and porosity of the soil has high percentage in the deltaic environment that deposit homogeneous soil formations.

To monitor the rate of E.coli concentration in homogeneous soil formation, mathematical equations were developed through the influential variables in the system. Developed mathematical equations were derived and produced the model that will monitor E.coli migration in homogeneous soil formations, the model developed will be useful to experts in design and construction of ground water system in the deltaic environment, where homogeneous formations are deposited.

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